

Marks : 70

Time : 3 Hrs

Section A

Q.1 Select and write the correct answer.

16

- (i) If $p \vee q$ is true and $p \wedge q$ is false then which of the following is not true?
a) $p \vee q$ b) $p \leftrightarrow q$ c) $\sim p \vee \sim q$ d) $q \vee \sim q$
- (ii) The principal solution of $\tan x = \sqrt{3}$ is
a) $\frac{\pi}{6}, \frac{5\pi}{6}$ b) $\frac{\pi}{6}, \frac{\pi}{6}$ c) $\frac{\pi}{3}, \frac{4\pi}{3}$ d) $\frac{\pi}{3}, \frac{2\pi}{3}$
- (iii) The cosine of the angle between the planes $2x + 2y - 3z = 5$ and $x - 2y + 3z = 7$ is
a) $\frac{6}{\sqrt{238}}$ b) $\frac{11}{\sqrt{248}}$ c) $\frac{11}{\sqrt{238}}$ d) $\frac{12}{\sqrt{248}}$
- (iv) The value of 'a' if the vectors $\bar{i} - 2\bar{j} + \bar{k}$, $a\bar{i} - 5\bar{j} + 3\bar{k}$ and $5\bar{i} - 9\bar{j} + 4\bar{k}$ are coplanar is
a) 1 b) 2 c) 3 d) 4
- (v) If $y = \sqrt{\log x} + \sqrt{\log x} + \sqrt{\log x} + \dots \infty$, then $\frac{dy}{dx} =$ —
a) $\frac{x}{2y-1}$ b) $\frac{x}{2y+1}$ c) $\frac{1}{x(2y-1)}$ d) $\frac{1}{x(1-2y)}$
- (vi) —The value of $\sqrt{16.1}$ is
a) 4.125 b) 4.25 c) 4.0125 d) 4.1
- (vii) $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$
a) $\frac{\pi}{2}$ b) $\frac{\pi}{2} + 1$ c) $\frac{\pi}{2} - 1$ d) $\frac{\pi}{2} + 2$
- (viii) The solution of the differential equation $e^{\frac{dy}{dx}} = x$ is
a) $y = x \log x + x + c$ b) $y = x \log x - x + c$
c) $y = \frac{1}{x} + c$ d) $y = x - x \log x + c$

Q.2 Answer the following.

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- (i) Find the Polar co-ordinates of a point whose cartesian co-ordinates are $(-\sqrt{3}, 1)$
- (ii) Find the joint equation of two lines passing through the origin and having slopes 2 and -3.
- (iii) Find $\frac{dy}{dx}$ if $y = [a \cos^3 x + b \sin^3 x]^3$
- (iv) Find the Order and degree of the differential equation

$$\sqrt{\left(\frac{dy}{dx}\right)^3} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}}$$

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Section B

Attempt any Eight.

16

- Q.3** Write the negations of (a) Some natural numbers are not integers.
(b) $\forall n \in N, n + 1 > 2$
- Q.4** Solve the equations using Reduction method:
 $x + y + z = 6; 3x - y + 3z = 10; 5x + 5y - 4z = 3$
- Q.5** Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$
- Q.6** Find 'k' if $2x + y = 0$ is one of the lines given by $3x^2 + kxy + 2y^2 = 0$
- Q.7** The Cartesian equation of a line is $3x + 1 = 6y - 2 = 1 - z$. Find its vector equation.
- Q.8** Find the general solution: $\cos 3x = \frac{1}{\sqrt{2}}$
- Q.9** Find the values of 'x' if $f(x) = 2x^3 - 15x^2 - 84x - 7$ is a decreasing function.
- Q.10** Evaluate: $\int \frac{dx}{1+3\sin^2 x}$
- Q.11** For the following probability distribution of X
- | | | | | | |
|--------|----|----|----|----|----|
| X | 0 | 1 | 2 | 3 | 4 |
| P(X=x) | 2k | 5k | 4k | 3k | 2k |
- find (i) 'k' (ii) $P(X \geq 2)$ s
- Q.12** Find the maximum volume of a right circular cylinder, if the sum of its radius and height is 6 units.
- Q.13** Form a differential equation by eliminating the arbitrary constants A and B: $y = Ae^{4x} + Be^{-4x}$
- Q.14** A fair coin is tossed 6 times. Find the probability that it shows tail exactly 2 times.

Section C

Attempt any Eight.

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- Q.15** In ΔABC , $a = 4$, $b = 5$, $c = 3$, find the value of (a) $\sin \frac{A}{2}$ (b) $\cos A$ and (c) $A(\Delta ABC)$
- Q.16** Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 10$ contain the sides of an equilateral triangle. Find the area of the triangle.
- Q.17** Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 4, -1)$ on the line $\frac{x-1}{2} = \frac{y}{1} = \frac{z}{2}$.
- Q.18** Prove using vector methods: "The medians of a triangle are concurrent".
- Q.19** Find the direction ratios of a vector perpendicular to the two lines having direction ratios $-2, 1, -1$ and $-3, -4, 1$.
- Q.20** Find the vector equation of a plane passing through the points $(2, 3, 1)$, $(4, -5, 3)$ and parallel to X-axis.
- Q.21** If $x = a \cos^3 t$, $y = a \sin^3 t$, show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$
- Q.22** Evaluate: $\int \frac{1}{3+2\sin x + \cos x} dx$

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- Q.23** The slope of the tangent to the curve at any point is equal to $y + 2x$. Find the equation of the curve passing through the origin.
- Q.24** A spherical snow ball is melting so that its volume is decreasing at the rate of 8cc/sec. Find the rate at which its radius is decreasing when it is 2cm. Also find the rate of change of surface area at that time.
- Q.25** Let $X \sim B(n, p)$
 (i) If $n = 15$, $E(X) = 5$, find 'p' and $\text{Var}(X)$
 (ii) If $E(X) = 9$, $\text{Var}(x) = 4.5$ find n and p.
- Q.26** Obtain the expected value and Variance of X for the following probability distribution.

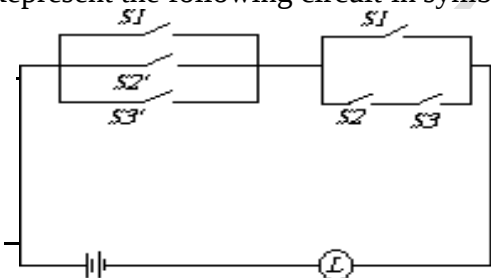
$X = x$	0	1	2	3	4
$P(X = x)$	0.2	0.1	0.2	0.4	0.1

Section D

Attempt any five.

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- Q.27** Represent the following circuit in symbolic form and construct its input output table.



- Q.28** Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ using Adjoint method.
- Q.29** If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points A, B, C w.r.t. origin and point C divides AB internally in the ratio of m:n, then prove that $\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$. Hence find the position vector of the point C which divides AB, A(2, -1, 3) and B(-5, 2, -5) internally in the ratio of 3:2
- Q.30** Minimize $z = 30x + 20y$ subject to $x + y \leq 8, x + 4y \geq 12, 5x + 8y \geq 20, x, y \geq 0$
- Q.31** Prove: If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x then $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. Also, find $\frac{dy}{dx}$ if $y = \sin^2(x + 3)$
- Q.32** Evaluate: $\int e^{3x} \sin 2x \, dx$
- Q.33** Prove: $\int_{-a}^a f(x) \, dx = 0$ if f is odd
 $= 2 \int_0^a f(x) \, dx$ if f is even.
- Q.34** Find the area of the region in the first quadrant bounded by the circle $x^2 + y^2 = 4$, X-axis and the line $x = y\sqrt{3}$.